We Don't Quite Know What We Are Talking About

When we talk about volatility.

Daniel G. Goldstein and Nassim Nicholas Taleb

There is no particular normative reason to express or measure volatility in any one of the several ways possible, provided we remain consistent. Once we express it in a particular way, however, substituting one measure for another will lead to a consequential mistake.

Suppose we measure volatility in root mean square deviations from the mean, as in conventional statistics. It would then be an error to substitute a definition and consider it mean deviation in the course of decision-making, opinion formation, or descriptions of the property of the process. Yet people do make this mistake.

This brief note provides experimental evidence that participants with varied backgrounds in financial markets err in interpreting a physical linear description (in mean absolute returns per day) as a calculated non-linear measure (standard deviation). We illustrate the confusion, and discuss its implications for financial decision-making and portfolio risk management.

EXPERIMENTS

To investigate the common understanding of mean absolute deviation, we asked professionals and students of finance the question:

> A stock (or a fund) has an average return of 0%. It moves on average 1% a day in absolute value; the average up move is 1%, and the average down move is 1%. This does not mean that all up moves are 1%—some are 0.6%, others 1.45%, and so on. Assume that we live in the Gaussian world where the returns (or daily percentage moves) can be safely

Daniel G. Goldstein
is an assistant professor of Marketing at London Business School, London, U.K.
dgoldstein@london.edu

Nassim Nicholas Taleb
is a visiting professor at London Business School, London, U.K.

modeled using a normal distribution. Assume that a year has 256 business days. Our questions concern the standard deviation of returns (i.e., of the percentage moves), the sigma that is used for volatility in financial applications. What is the daily sigma? What is the yearly sigma?

Our suspicion that there would be considerable confusion is fed by years of hearing options traders make statements like "an instrument that has a daily standard deviation of 1% should move 1% a day on average." Not so. In the Gaussian world, where x is a random variable, assuming a mean of 0, in expectation, the ratio of standard deviation to mean deviation should satisfy the equality:

$$\frac{\sum |x|}{\sqrt{\sum x^2}} = \sqrt{\frac{2}{\pi}}$$

Since mean absolute deviation is about 0.8 times the standard deviation, in our problem the daily sigma should be 1.25%, and the yearly sigma should be 20.00% (which is the daily sigma annualized by multiplying by 16, the square root of the number of business days).

To test the hypothesis that mean absolute deviation is confused with standard deviation, we ran studies with three groups: 1) 97 portfolio managers, assistant portfolio managers, and analysts at investment management companies who were taking part in a professional seminar; 2) 13 Ivy League graduate students preparing for a career in financial engineering; and 3) 16 investment professionals working for a major bank. The question was presented in writing and explained orally to make sure definitions were clear.

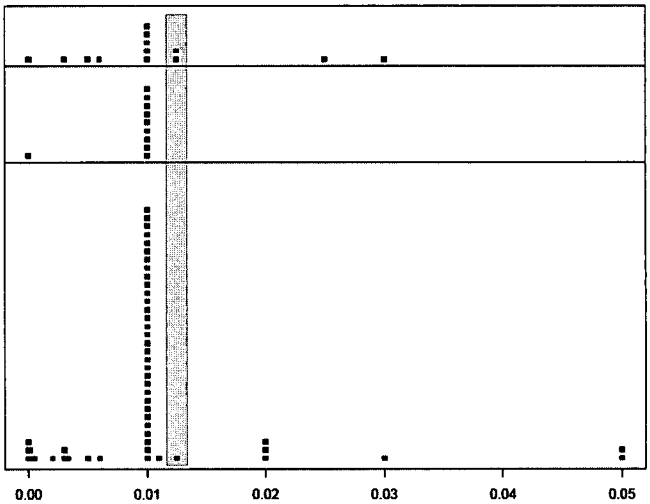
All respondents in the latter two groups turned in responses, compared to 58 in the first group. One might expect this sort of self-selection to improve accuracy.

RESULTS

Exhibit 1 shows frequency histograms of responses to the daily sigma question, converted to decimal notation. Only 3 of the 87 respondents arrived at the correct answer of 0.0125. The modal answer of 0.01 made up more than half of the responses received. Nine people gave responses of blanks or question marks. Far from symmetrical, the ratio of underestimations of volatility to overestimations was 65 to 10.

Performance for yearly sigmas was even worse. No one submitted a correct response. Here, the modal answer was 0.16, which appears to be the correct annualization of the incorrect daily volatility. Eleven people responded with blanks or question marks. The ratio of underestimations to overestimations was 76 to 9.

EXHIBIT 1
Estimates of Daily Standard Deviation



The top plot represents the responses of investment professionals at a major bank. The middle plot represents responses of graduate students in financial engineering (excluding one outlier at 0.1). The bottom plot represents responses of professional portfolio managers and analysts. The correct answer under the stated Gaussian assumptions (0.0125) is shaded in gray.

CONCLUSION

The error we point out is more consequential than it seems. The dominant response of 1% shown in the Exhibit suggests that even financially and mathematically savvy decision-makers treat mean absolute deviation and standard deviation as the same thing. Although a Gaussian random variable that has a daily percentage move in absolute terms of 1.00% has a standard deviation of about 1.25%, it can reach up to 1.90% in empirical distributions (emerging market currencies and bonds). Mean absolute deviation is by Jensen's inequality lower than (or equal to) standard deviation.*

In a world of fat tails, the bias increases dramatically. Consider the vector of dimension 10^6, composed of 999,999 elements of 0 and a single one of 10^6: $V = \{0,0,0,0,\ldots,10^6\}$. Here the standard deviation would be 1,000 times the average move.

For a Student-t with 3 degrees of freedom (often used to model returns in financial markets in value at risk simulations), standard deviation is 1.57 times mean deviation:

$$\frac{\sum |x|}{\sqrt{\sum x^2}} = \frac{2}{\pi}$$

See Bouchaud and Potters [2003] and Glasserman, Heidelberger, and Shahabuddin [2002].

Conversations with our respondents revealed that they rarely had an immediate understanding of the error when we pointed it out. Yet when we asked them to present the equation for "standard deviation," they expressed it flawlessly as the root mean square of deviations from the mean. Whatever the respondents' reason for the error, it did not result from ignorance of the concept. Indeed, most participants would have failed a basic statistics course had they not been aware of the mathematical definition, but when given data that are clearly not a standard deviation, they treat it as one.

Kahneman and Frederick [2002] discuss a similar problem: that statisticians make basic statistical mistakes outside the classroom:

> The mathematical psychologists who participated in the survey not only should have known better—they did know better . . . . Most of them would have computed the correct answers on the back of an envelope.

Why is this problem relevant? This sloppiness in translation between mathematics and applications can have severe effects, considering that practitioners speak with co-workers, customers, and the media about volatility on a regular basis. We know of instances in the financial media where journalists make the same mistake in explaining the volatility index VIX to the general public.

Either we have the wrong intuition about the right volatility, or the right intuition but the measure of volatility is the wrong one. Two roads lead out of this unfortunate situation. Either we can continue defining volatility as we do, and conduct further research to see if the error can be made to disappear with training, or we can do as the probabilists of the Enlightenment age did. When the intuitions of *hommes éclairés* did not align with the valuations of the expected value formula, it was the mathematicians who dreamed up something more intuitive, namely, expected utility (see Daston [1988]).

Perhaps some day finance will adopt a more natural metric than standard deviation. Until then, users should rely on definitions, not intuitions, where volatility is concerned.

ENDNOTE

*More technically, the norm $L^p = \left(\frac{1}{n}\Sigma |x|^p\right)^{1/p}$ increases with p.

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